# Viscosity of Vinyl Aromatic Polymer Solutions 

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#### Abstract

$V$ iscosity data are presented for benzene solutions of three vinyl aromatic polymers: poly(1-vinylnaphthalene), poly(2-vinylnaphthalene) and poly(vinylhiphenyl) at $30^{\circ} \mathrm{C}$ and in the range of concentration $c \leqslant 4 /[\eta]$. The results are compared with those for polystyrene in a theta and a good solvent. The applicability of the superposition scheme developed earlier and the molecular weight dependence of the reducing parameters are examined. For reduced viscosities $\tilde{\eta} \equiv \eta_{\mathrm{sp}} / \mathrm{c}[\eta]$ below about $1 \cdot 6$, superposition is achieved for all molecular weights. Observed differences in the molecular weight range of validity of superposition at elevated concentrations for the systems considered can be correlated with chain stiffness and solvent-polymer interactions, as characterized by the parameter $B$. A new reduced variable $\tilde{c_{1}}$ is suggested for a comparison of the relative positions of the generalized viscosity functions, $\tilde{\eta}=\mathrm{f}\left(\tilde{c}_{1}\right)$, of different polymer solutions. The relative positions of the curves appear to be related to the relative values of the long range interaction parameter $B$.


It has been shown that the viscosities of polymer solutions can be represented by general functions, independent of molecular weight, $M$

$$
\begin{equation*}
\eta_{\mathrm{sp}} / c[\eta] \equiv \tilde{\eta}(\tilde{c}) \tag{1}
\end{equation*}
$$

where $\tilde{c} \equiv c / \gamma^{\prime}$, and the concentration parameter $\gamma^{\prime}=\gamma^{\prime}(M, T)$ is characteristic of the polymer-solvent system and independent of concentration $c$.

The above statement implies a principle of corresponding states and was first derived from an analysis of $\eta / c$ data of polystyrene (PS) solutions in good and theta solvents ${ }^{1}$. Later the principle was found to hold for PS-near-theta-solvent systems ${ }^{2}$ and very recently proved to be valid for polyisobutene (PIB), polyvinyl chloride (PVC), polyvinyl alcohol, cellulose derivatives and polyisoprene solutions ${ }^{3}$. The viscosities for these twelve different polymer-solvent systems were measured in different laboratories, by different methods using samples of different heterogeneities. The representation proposed gave very good results in the whole range of concentrations (up to 50 per cent) for polymer-poor solvent systems. With increasing solvent power deviations in the lower molecular weight region were observed. These systems show analogous deviations in respect of their $[\eta] / M$ and $[\eta] / T$ relations. In the PS-toluene system we observed deviations for $M \leqslant 15000$, in PIB-isooctane for $M \leqslant 20000$, and in PVCcyclohexanone for $M \leqslant 26000$.

In this paper we continue our investigations of concentrated systems and present $\eta / c$ data for benzene solutions at $30^{\circ} \mathrm{C}$ for three vinyl aromatic polymers (PVAr), namely poly(4-vinylbiphenyl) (PVB), poly(1-vinylnaphthalene) ( $\mathrm{P} \mid \mathrm{VN}$ ), and poly(2-vinylnaphthalene) ( P 2 VN ), in the range of

[^0]concentrations $c \leqslant 4 /[\eta](\mathrm{g} / \mathrm{d})$ ). The $\eta / c$ relations for these polymers can be compared directly with those for the PS-solvent systems'. It is of particular interest to examine how the increased bulkiness of these aromatic side groups affects the polymer chain configuration, the solution behaviour and the range of validity of the general equation (1).

## EXPERIMENTAL

The polymers were synthesized for the Jet Propulsion Laboratory by Dr J. Heller at Stanford Research Institute and received through the courtesy of Dr J. Moacanin. They were prepared by anionic polymerization, fractionated and characterized by gel permeation chromatography ${ }^{1}$ (GPC), which was calibrated using light scattering and osmotic data at the JPL. PVB and P2VN samples were reprecipitated in our laboratory from benzene solution by methyl alcohol and dried in a vacuum oven for two weeks at $50^{\circ} \mathrm{C}$. Selected samples of these polymers measured before and after the precipitation showed no change in $\left[\eta\right.$ ] and in the Huggins parameter $k_{1}$. For P1VN we found that additional purification was not needed. The characteristics of all samples are presented in Table 1 .

Table 1. Characteristics of polymer samples

| No. | Symbol | $M_{w} \times 10^{-3}$ | $M_{w} / M_{n}$ | [ $\eta$ ]* | $\begin{gathered} \text { Benzene } \\ k_{1}{ }^{*} \\ \hline \end{gathered}$ | $\begin{aligned} & --30^{\circ} \mathrm{C} \\ & \left(k_{1}[\eta]\right)^{-1} \end{aligned}$ | $\gamma^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Poly(1-vinylnaphthalene) |  |  |  |  |  |  |  |
| 1 | P1VN-59 | 155.3 | $1 \cdot 14$ | 0.387 | 0.43 | 6.01 | 1.00 |
| 2 | P1VN-66 | $147 \cdot 3$ | $1 \cdot 48$ | $0 \cdot 372$ | $0 \cdot 40$ | 6.72 | 1.03 |
| 3 | P1VN-88 | $103 \cdot 4$ | 1.05 | 0.270 | 0.46 | 8.05 | 1.31 |
| 4 | P1VN-8036 | 52.2 | 1.06 | 0.156 | 0.50 | 12.82 | $1 \cdot 69$ |
| Poly(2-vinylnaphthalene) |  |  |  |  |  |  |  |
| 5 | P2VN-44 | 736.1 | $1 \cdot 55$ | 1.072 | 0.33 | $2 \cdot 83$ | 0.555 |
| 6 | P2VN-70 | $563 \cdot 4$ | 1.88 | 0.880 | 0.36 | $3 \cdot 16$ | 0.660 |
| 7 | P2VN-58 | $306 \cdot 1$ | $1 \cdot 41$ | 0.662 | $0 \cdot 36$ | $4 \cdot 20$ | 0.833 |
| 8 | P2VN-46 | 182.5 | $1 \cdot 29$ | $0 \cdot 453$ | 0.42 | $5 \cdot 26$ | 1.00 |
| 9 | P2VN-68 | 64.0 | - | 0.229 | 0.49 | $8 \cdot 91$ | 1.72 |
| 10 | P2VN-61 | 48.9 | - | $0 \cdot 204$ | 0.50 | $9 \cdot 80$ | $2 \cdot 02$ |
| Poly(4-vinylbiphenyl) |  |  |  |  |  |  |  |
| 11 | PVB-63 | 1104.0 | $2 \cdot 15$ | 1.281 | 0.36 | $2 \cdot 17$ | 0.312 |
| 12 | PVB-41 | $706 \cdot 5$ | 2.06 | 0.940 | $0 \cdot 33$ | $3 \cdot 22$ | $0 \cdot 468$ |
| 13 | PVB-71 | 169.4 | 1.45 | 0.381 | 0.33 | 7.95 | 1.00 |
| 14 | PVB-56 | 103.0 | 1.52 | 0.248 | $0 \cdot 40$ | 10.08 | $1 \cdot 41$ |
| 15 | PVB-51 | $80 \cdot 8$ | 1.26 | 0.234 | 0.36 | 11.87 | 1.52 |
| 16 | PVB-8036 | $10.5 \dagger$ | 12 | 0.070 | 0.85 | 16.83 | 2.52 |

* Average of the values shown in Table 3.
$\dagger$ Ref. 4.
Mallinckrodt's analytical reagent grade thiophene-free benzene was further purified by rectification on a packed column over sodium wire. For all the experiments the middle fraction (b.pt $=80.0$ to $80.1^{\circ} \mathrm{C}$ ) from the same batch was used.
The polymer solutions were prepared one day before use in 10 ml volumetric flasks by heating the polymer-solvent mixture up to $50^{\circ} \mathrm{C}$ for $c a .30$ minutes. Due to special care taken during the reprecipitation of
polymer and the purification of the solvent, both components were dust-free and the solutions did not require filtration.
The viscosity measurements were carried out in a Hewlett-Packard Autoviscometer Model 5901B with constant temperature bath, Model 5910A, using Cannon-Ubbelohde dilution viscometers, calibrated by means of NBS standard viscosity oils. The viscometer constants are shown in Table 2. Kinetic energy and density corrections were applied to all individual measurements. We did not use any correction for a rate of shear dependence. From Table 2 one can see that for all systems $\beta \leqslant \beta_{\text {max }} \simeq \beta_{\text {ert. }} / 10$, where $\beta_{\text {ert. }}$ is the characteristic quantity of shear above which $[\eta]$ starts measurably to decrease ${ }^{5}$. Thus we should not expect any deviation due to shear stress in the region of infinite dilution. As far as the higher range of $c$ and $\eta$ is concerned, the viscosity of the same solution ( $\eta \simeq 1.5 \mathrm{cP}$ ) measured in either viscometer 1 or 2 had the same value within experimental error, indicating the absence of shear stress influence in this region as well.

Table 2. Characteristics of viscometers

| No. | Cannon Inst. Co. <br> Code | $\bar{A}$ | $\bar{B}$ | $G_{\text {max. }} \times 10^{-3} / \mathrm{sec}$ | $\beta_{\text {max. }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{D}-707 / 50$ | 0.0041543 | 1.425 | 1.48 | 0.0261 |
| 2 | $\mathrm{D}-442 / 100$ | 0.013718 | 2.60 | 0.89 | 0.0158 |

Constants $\bar{A}$ and $\bar{B}$ from the relation $v(\mathrm{eSt})=\vec{A}_{\tau}-\vec{B} / \tau$, where $\tau$ is the efflux time in seconds, $G_{\text {max. }}$ and $\beta_{\text {max }}$-maximum rate of shear and of the characteristic parameter of shear: $\beta_{\text {max. }} \equiv\left([\eta] \eta_{0} M / R T\right) \dot{G}_{\text {max }}$. ( $\eta_{0}$ is solvent viscosity).

For each viscosity/concentration curve at a given molecular weight, four initial solutions were usually prepared and these solutions subsequently diluted in the viscometer. The $[\eta] \mathrm{s}$ were determined from separate sets of highly diluted initial solutions. To calculate each $[\eta$ ] value two methods were applied: (1) using the viscometer constants $\bar{A}$ and $\bar{B}$ (Table 2) and the solution density, the viscosity of each solution was computed from its efflux time, $\tau$. Using the solvent viscosity, $\eta_{0}=0.564 \mathrm{cP}$, a plot of ( $\eta_{\mathrm{sc}} / c$ ) versus $c$ was constructed. From this plot $[\eta]$ and $k_{1}$ were obtained directly. These values are shown in columns 3 and 4 of Table 3. (2) The second way of calculating $[\eta]$ had been proposed by one of the authors ${ }^{6}$ : instead of applying the kinetic and density corrections to each individual $\tau$ value, the set of constants $[\tau]$ and $\dot{k}_{17}$ was calculated from the $\left(\tau-\tau_{0}\right) /\left(\tau_{c} c\right)$ versus $c$ plot ( $\tau_{0}$ is the efflux time of the solvent). From the values of $[\tau]$ and $k_{17},[\eta]$ and $k_{1}$ were calculated using the following equations:

$$
\begin{gather*}
{[\eta]=[\tau](1+\boldsymbol{H}) /(1-\boldsymbol{H})+\boldsymbol{D}} \\
k_{1}=k_{1 \tau}(1-H) /(1+H)-H(1-H) /(1+H)^{2}+[\eta] D \tag{2}
\end{gather*}
$$

where $H \equiv \bar{B} /\left(\bar{A} \tau_{0}^{2}\right)$ and $D=10^{-2}\left(d_{0}^{-1}-d_{p}^{-1}\right)$. As usual $[\eta]$ is expressed in $\mathrm{dl} / \mathrm{g}$ units, and $d_{0}$ and $d_{p}$ are solvent and polymer densities in $\mathrm{g} / \mathrm{ml}$.

In columns 5 and 6 (Table 3) the [ $\tau]$ and $k_{1 \tau}$ data are shown and in columns 7 and $8[\eta]$ and $k_{1}$ calculated from these by equation (2). As can be seen, the agreement is quite good. The [ $\eta$ ] and $k_{1}$ data shown in Table 1 are the averages of those shown in Table 3.

Table 3. [ $\eta$ ] and $k_{1}$ for vinyl aromatic polymers

| No. | Polymer | $\begin{gathered} {[\eta]} \\ (\text { from } \eta) \end{gathered}$ | $\begin{gathered} k_{1} \\ \text { (from } \eta \text { ) } \end{gathered}$ | $\begin{gathered} {[\tau]} \\ (\text { from } \tau) \end{gathered}$ | $\begin{gathered} k_{1 \tau} \\ \text { (from } \tau) \end{gathered}$ | $\begin{gathered} {[\eta]} \\ e q .(2) \end{gathered}$ | $\begin{gathered} k_{1} \\ e q .(2) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | P1VN-59 | $0 \cdot 387$ | 0.42 | $0 \cdot 371$ | 0.46 | $0 \cdot 386$ | 0.43 |
| 2 | P1VN-66 | $0 \cdot 373$ | $0 \cdot 40$ | $0 \cdot 356$ | 0.43 | $0 \cdot 370$ | 0.40 |
| 3 | P1VN-88 | $0 \cdot 272$ | 0.44 | $0 \cdot 257$ | $0 \cdot 51$ | $0 \cdot 267$ | $0 \cdot 48$ |
| 4 | P1VN-8036 | $0 \cdot 157$ | $0 \cdot 48$ | $0 \cdot 148$ | $0 \cdot 55$ | 0.155 | $0 \cdot 52$ |
| 5 | P2VN-44 | 1.072 | $0 \cdot 32$ | 1.041 | 0.36 | 1.073 | $0 \cdot 33$ |
| 6 | P2VN-70. | $0 \cdot 880$ | $0 \cdot 37$ | 0.854 | $0 \cdot 37$ | 0.880 | 0.35 |
| 7 | P2VN-58 | $0 \cdot 663$ | $0 \cdot 36$ | $0 \cdot 639$ | $0 \cdot 39$ | $0 \cdot 660$ | $0 \cdot 36$ |
| 8 | P2VN-48 | 0.453 | 0.42 | 0.433 | 0.45 | $0 \cdot 448$ | 0.43 |
| 9 | P2VN-68 | $0 \cdot 230$ | 0.49 | $0 \cdot 219$ | 0.52 | $0 \cdot 228$ | 0.49 |
| 10 | P2VN-61 | $0 \cdot 206$ | 0. 50 | 0-194 | 0.53 | $0 \cdot 203$ | $0 \cdot 50$ |
| 11 | PVB-63 | $1 \cdot 282$ | $0 \cdot 36$ | $1 \cdot 247$ | $0 \cdot 38$ | $1 \cdot 280$ | $0 \cdot 36$ |
| 12 | PVB-41 | 0.940 | 0.32 | $0 \cdot 914$ | $0 \cdot 36$ | 0.939 | $0 \cdot 34$ |
| 13 | PVB-71 | $0 \cdot 380$ | $0 \cdot 36$ | $0 \cdot 370$ | $0 \cdot 33$ | 0.383 | $0 \cdot 31$ |
| 14 | PVB-56 | $0 \cdot 250$ | $0 \cdot 38$ | $0 \cdot 237$ | 0.44 | 0. 246 | 0.41 |
| 15 | PVB-51 | $0 \cdot 235$ | $0 \cdot 36$ | $0 \cdot 224$ | $0 \cdot 39$ | 0.233 | $0 \cdot 36$ |
| 16 | PVB-8036 | $0 \cdot 070$ | $0 \cdot 84$ | 0.064 | $0 \cdot 90$ | 0.070 | 0.86 |

RESULTS
The $\eta_{\text {g® }} / c$ values obtained for the three polymers are exhibited in Table 4. The $\widetilde{\eta}$ [equation (1)] calculated from these data were plotted against $c$ on a double logarithmic plot, and the curves obtained for the various $M_{w}$ samples of each polymer were shifted along the $c$ axis $^{1}$ to obtain the master curve of $\tilde{\eta}=\tilde{\eta}(\tilde{c})$ as shown in Figures 1 to 3. By this procedure values of


Figure 1 -Reduced viscosity quantity $\tilde{\eta}$ as a function of reduced concentration $\tilde{\boldsymbol{c}}$ for P1VN in benzerie at $30^{\circ} \mathrm{C}$. Insert: [ $\eta$ ], reducing concentration factors $\gamma_{1}$ and $\gamma_{2}$ as functions of $M_{w}$

Table 4．Viscosities of benzene solutions of vinyl aromatic polymers at $30^{\circ} \mathrm{C}$

| No． | $\underset{\mathrm{g} / \mathrm{dl}}{c}$ | $\begin{aligned} & \eta_{\mathrm{sp} / \mathrm{p}} \\ & \mathrm{di} / \mathrm{g} \end{aligned}$ | $\underset{\mathrm{g} / \mathrm{d} \mathrm{~d}}{c}$ | $\begin{aligned} & n_{\mathrm{si}} / c \\ & \mathrm{di} / \mathrm{g} \end{aligned}$ | $\underset{g / d 1}{c}$ | $\begin{gathered} n_{\mathrm{sd}} / \bar{c} / \mathrm{c} \\ \mathrm{~d} / / \mathrm{g} \end{gathered}$ | $\underset{\mathrm{g} / \mathrm{dl}}{c}$ | $\begin{aligned} & \eta_{\mathrm{sp}} / c \\ & \mathrm{dpl/g} \end{aligned}$ | $\underset{g / \mathrm{d} 1}{c}$ | $\begin{aligned} & n_{\mathrm{si}} / c \\ & \mathrm{di} / \mathrm{g} \end{aligned}$ | $\underset{\mathrm{g} / \mathrm{d} 1}{c}$ | $\begin{aligned} & n_{\mathrm{g} /} / c \\ & \mathrm{di} / \mathrm{g} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Poly（1－vinylnaphthalene） |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $M_{w}=52000$ |  | $M_{w}=103400$ |  | $M_{\text {w }}=147300$ |  | $M_{w}=155300$ |  |  |  |  |  |
| $\frac{1}{2}$ | 0.4380 0.5475 | 0.1628 0.1640 | 0.3221 0.4920 | 0.2822 0.2878 | $\begin{aligned} & 0.3514 \\ & 0.4838 \end{aligned}$ | $\begin{aligned} & 0.3906 \\ & 0.3987 \end{aligned}$ | $\begin{gathered} 0.5009 \\ 0.7927 \end{gathered}$ | $\begin{aligned} & 0.4196 \\ & 0.4375 \end{aligned}$ |  |  |  |  |
| 3 | 0.7181 | 0.1669 | 0.7170 | 0.2947 | 0.6594 | ${ }_{0}^{0.4083}$ | 1.2897 | 0.4689 |  |  |  |  |
| $\stackrel{4}{5}$ | 0.9382 1.2259 | 016880171 | ${ }_{0}^{0.7493}$ | $\xrightarrow{0.2979}$ | 0．6737 | ${ }_{0}^{0.4088}$ | 1.3207 2.2514 | 0.4735 0.5325 |  |  |  |  |
| 6 | 1.5544 1.159 | 0.1748 0.1839 | 1.0540 | 0.3083 0.323 | 0．8928 | 0.420 0.4325 | 2． 5767 | 0．5534 |  |  |  |  |
| 8 | 2．6342 | ${ }_{0}^{0} 1907$ | ${ }_{2}$ ． 0432 | ${ }_{0} .3399$ | 1.1804 | ${ }_{0}^{0.4309}$ | 3.9412 | － 0.6378 |  |  |  |  |
| 9 | 3.3572 4.1486 | ${ }_{0}^{0.2003}$ | 2.7214 <br> 4.358 <br> 1 | － $\begin{aligned} & 0.3626 \\ & 0.4024\end{aligned}$ | 1．2595 | 0.4408 0.4602 | 4． 51900 | 0.6576 0.7870 |  |  |  |  |
| 11 | ${ }_{5} 5.7682$ | 0.2384 | 5．5294 | 0.4591 | 1.7515 | 0.4641 | 5.9732 | 0.8300 |  |  |  |  |
| 12 | 7.6550 10.167 | 0.2689 0.327 | ${ }_{\text {7．2494 }}^{10} \mathbf{1}$ | ${ }_{\substack{0.5635 \\ 0.7357}}^{0.5}$ | 2．9249 | 0.5377 0.5630 | 7．3462 | 1．0009 |  |  |  |  |
| 14 | 14．426 | 0．4564 | 12.276 | ${ }_{0} 8954$ | 4.0981 | ${ }_{0}^{0.6109}$ | ${ }_{10}^{8} \cdot 225$ | ${ }_{1}^{1.3612}$ |  |  |  |  |
| 15 | 18.740 22 2 | 0.6855 0.923 0.9203 |  |  | 4．7432 | ${ }_{0}^{0.6512}$ | 14．516 | －${ }_{\text {1．}}^{1.1923}$ |  |  |  |  |
| 17 | 26.528 | 1.3973 |  |  | 6.5882 | 0.8143 | － |  |  |  |  |  |
| 18 | 二 |  |  |  | 7．5686 | ${ }_{1}^{0} 1.0350$ | 二 |  |  |  |  |  |
| 20 |  |  |  |  | 8．9162 | 1．1295 |  |  |  |  |  |  |
| ${ }_{22}^{21}$ |  |  |  |  | 11.418 15.414 | 1．4566 2.3697 | － |  |  |  |  |  |
|  | Poly（2－vinylnaphthalene） |  |  |  |  |  |  |  |  |  |  |  |
|  | $M_{w}=48900$ |  | $M_{w w}=64000$ |  | $M_{w}=182500$ |  | $M_{w}=306100$ |  | $M_{w}=563400$ |  | $M_{w}=736100$ |  |
| $\frac{1}{2}$ | 0.6440 0.9056 | 0.2192 0.224 | 0.6245 0.7780 | 0.2464 0.2505 | 0.3493 0.6036 | 0.4834 0.5040 | － $\begin{aligned} & 0.2374 \\ & 0.4482\end{aligned}$ | ${ }_{0}^{0.7046}$ | 0.2538 0.4242 | 0.9512 1.0004 | 0.2495 0.4094 | 1.1731 1.2221 |
| 3 | 1.3221 | ${ }_{0}^{0} 2326$ | 1.2298 | ${ }_{0}^{0624}$ | ${ }_{0} 8422$ | ${ }_{0}^{0.5297}$ | 0.6437 | ${ }_{0}^{0} .7656$ | ${ }_{0} .4428$ | 1.0358 | ${ }_{0}^{0.4148}$ | ． 2407 |
| 4 | 1.7840 1.4546 | ${ }_{0}^{0.2419}$ | 1．6969 | 0.2705 0.278 | 0．8969 | ${ }_{0}^{0.5314}$ | 0.6984 0.8636 | 0.7594 0.8007 | 0．6217 | 1.0598 | ${ }_{0}^{0.5289}$ | ． 2724 |
| 5 | 2．44498 | 0.2545 0.2725 | ${ }_{2.0461}^{1.6700}$ | 0.2728 0.2834 | 1.1554 1.2098 | 0.5578 0.5560 | － 1.8313 | 0.8007 0.8140 | ${ }_{0}^{0.7882}$ | ${ }_{1}^{1} 1.1235$ | ${ }_{0}^{0.8784}$ |  |
| －7 | － $\begin{aligned} & 4.9392 \\ & 5\end{aligned}$ | 0.3031 0.373 | 2．7347 | 0.3001 0.366 | 1．3954 | 0.5800 0.6317 | 1．2415 | 0.8590 0.8588 | 1．1362 | 1．2092 | 0.9014 <br> 1.085 | （1．4330 |
| ${ }_{9}^{8}$ | ＋5．934 | － 0 | 5.0589 | 0.3366 0.3703 | 2.8604 | ${ }_{0}^{0.7103}$ | 1.7091 | －0．8338 | ${ }_{2}$ | 1． 5826 1 | ${ }_{1}^{1.5927}$ |  |
| 10 | 14.498 | 0.5946 |  |  | ${ }^{4}$ 4． 19211 | ${ }_{\substack{0 \\ 1.14372}}^{1.682}$ | 2． 2.5170 | 2．3057 | 2．7593 | 1．6704 | 1．5927 | ${ }_{1}^{1.7115}$ |
| 11 12 | － | ＝ |  | － | 6.311 <br> 10.395 | （1．8922 | 2．5843 | 1. | 3.5225 4.4374 | 2．0179 | 2．0647 | 2． 2.43315 |
| 13 |  |  |  |  |  |  | 3.7891 4.6772 | 1．3246 | 5.4777 | 2.9102 | ${ }_{5}^{4.9243}$ | 3.2731 4.6467 |
| 15 | － | － |  |  |  |  | 5．7134 | 1.8534 | 二 |  |  |  |
| 16 | － | － | － | － | － |  | 9.3102 | 3．0244 | － | － | － |  |
|  | （Poly 4－vinylbiphenyl） |  |  |  |  |  |  |  |  |  |  |  |
|  | $M_{w w}=10500$ |  | $M_{x}=80800$ |  | $M_{w}=103000$ |  | $M_{w}=169400$ |  | $M_{w}=706500$ |  | $M_{w}=1104000$ |  |
|  | 1．9661 3.6604 | 0．07843 | 0．8694 | ${ }_{0}^{0.2533}$ | － $\begin{aligned} & 0.3356 \\ & 0.4677\end{aligned}$ | 0.2570 0.2597 | 0.2938 0.5534 | 0.3939 0.4061 | 0.1124 0.1506 | 0.9788 0.9793 | 0.2004 0.2952 | ${ }_{1}^{1.4604}$ |
| 3 4 4 | $\underset{\substack{5.6546 \\ 8.2120}}{ }$ | 0．09458 0.1080 | lither | － $\begin{aligned} & 0.2793 \\ & 0.2903\end{aligned}$ | c． 0 cosi | 0．2632 | ${ }_{0}^{0.6751}$ | ． 0.42175 | O． 0.233 0.239 | 1.0020 | ${ }_{0}^{0} .3571$ | 1．4945 |
| $\stackrel{4}{5}$ | 8.2120 10.784 1 | 0．1250 | 3．7641 | 0.39014 0.318 | 0．7984 | 0.2668 0.290 | 0.7578 | ${ }_{0}^{0.4256}$ | 0.2479 0.479 | $\xrightarrow{1.0947}$ | － 0.43995 | 1.5488 1.691 |
| ${ }_{7}$ | － 15.205 | －${ }_{\text {0．}}^{0.1632}$ | 3.7148 4.6201 | 0.3168 0.3436 | 1．0643 | 0.2758 0.2827 | 1．0358 | 0.4396 0.4597 | 0．5268 | 1． 1.0945 | 0.7587 <br> 0.943 | 1.7312 <br> 1.8400 |
| 8 | 24．901 | $\stackrel{3}{0.286}$ | 4.7231 6 | ${ }_{0}^{0.4410}$ | ${ }_{1}^{1.3587}$ | ${ }_{0}^{0.3821}$ | 1．4226 | ${ }_{0}^{0.4898}$ | 0.7846 | 1．18701 | 0.9486 1.807 | 1－8400 |
| ${ }_{10}^{9}$ |  |  | 8.2061 10.405 | $\xrightarrow{0.4566} 0$ | 2．3518 | ${ }_{0}^{0.3112}$ | 2.5449 | 0．5292 | 0．2993 | 1．1962 | ${ }_{1}^{1} .89938$ | 2.3430 2.3819 |
| 11 |  |  | 13.391 16.085 | 0．7140 | 3.7920 | 0.3528 | 3．3918 | 0.5886 | 1.4943 | 1.3832 | 2.3155 | 2.6870 |
| 12 13 |  | 二 | 16.085 19.089 | $\xrightarrow{0} 1.22288$ | 4．9546 | 0.3917 0.4489 | 4．7061 | ${ }_{0}^{0.6519} 0$ | 1．7560 | 1.4828 1.7071 | 2．9928 | 2．7877 |
| 14 |  |  | 20.925 | 1.4872 | 6．6275 | 0.4519 | 6．6585 | 0.8030 | 2.3170 | 1.6572 | 3.6900 | 3.8596 |
| 15 |  |  |  |  | 7.685 | ${ }_{0}^{0.4897}$ | 6． 62685 <br> 8 | 0.8092 0.9635 | 2．9324 | 1．8582 | 4．0555 | 4．1615 |
| 17 |  |  | － |  | 9．2756 | 0.5739 |  | （1．1982 | 3．7595 | 2.1671 |  |  |
| 19 |  |  |  |  | 11．513 | ${ }_{0} 0.6995$ |  | 1.515 | 4．8344 | 2．6368 |  |  |
| 20 |  |  |  |  | 13．770 | 0．8648 |  | － | 5.9437 | 3．1392 |  |  |
| 22 |  |  |  |  |  |  |  |  | （8．1689 | 4．4842 |  |  |
| 23 | － | － | － | － | － | － | － | － | 11．123 | 7.4864 | － | － |

the shifting parameter $\gamma^{\prime}$ were obtained，using a separate reference molecular weight $M^{\prime}$ for each polymer series：i．e．$\gamma^{\prime}\left(M^{\prime}\right) \equiv 1$ ．The values of $\gamma^{\prime}$ are listed in column 8 of Table 1．In Table 5 we present the numerical values of the parameters $a, a_{1}, K$ and $K_{1}{ }^{\prime}$ defined by the relations

$$
\begin{equation*}
[\eta]=K M^{a} ; \quad \gamma^{\prime}=K_{1}^{\prime} M^{-a_{2}}=\left(M^{\prime} / M\right)^{a_{1}} \tag{3}
\end{equation*}
$$



Figure 2-Reduced viscosity quantity $\tilde{\eta}$ as a function of reduced concentration $\tilde{\boldsymbol{c}}$ for P2VN in benzene at $30^{\circ} \mathrm{C}$. Insert: [ $\eta$ ], reducing concentration faotors $y_{1}$ and $\gamma_{2}$ as functions of $M_{w}$


Figure 3-Reduced viscosity quantity $\tilde{\eta}$ as a function of reduced concentration $\tilde{\boldsymbol{c}}$ for PVB in benzene at $30^{\circ} \mathrm{C}$. Insert: [ $\eta$ ], reducing concentration factors $\gamma_{1}$ and $\gamma_{2}$ as functions of $M_{v}$

Choosing now a common reference molecular weight for all systems, $M_{1} \equiv 10^{5}$, and defining $\gamma_{1}\left(M_{1}\right) \equiv 1$, we have the analogous relation $\gamma_{1}=K_{1} M^{-a_{1}}$ with $K_{1}=M_{1}^{a_{1}}$. In contrast to $\gamma^{\prime}$ the $\gamma_{1}^{\prime}$ s for different polymersolvent systems can be compared with each other.

In the above relations the absolute values of $\gamma^{\prime}$ or $\gamma_{1}$ depend on the choice of the reference molecular weight. In the previous papers of the series ${ }^{1-3}$ we have shown that as a consequence of the superposition principle, a parameter $\gamma_{2}$ may be defined by the relation

$$
\begin{equation*}
\gamma_{2}=C /\left(k_{1}[\eta]\right) \tag{4}
\end{equation*}
$$

where the numerical value of the constant $C$ depends solely on the choice of the reference molecular weight for $\gamma_{2}=1$. Setting $\gamma_{1}=\gamma_{2}$ we compute the $C$ values for solutions of each polymer. The root mean square error in $C$ is five per cent. The average values of $C$ for PVAr and PS are shown in Table 5. Using these $C$ values the individual values of $\gamma_{2}$ can be calculated. Their molecular weight dependence is shown in the inserts of Figures 1 to 3 along with the molecular weight dependence of $\gamma_{1}$ and [ $\eta$ ]. As one can see, $\gamma_{1}$ and $\gamma_{2}$ are scattered at random along the lines corresponding to equation (3), indicating the validity of equation (4).

## DISCUSSION

(1) Applicability of superposition principle

We define $M_{\text {crit. }}$ as the lowest molecular weight of a given polymer-solvent system, for which a single reduced function $\tilde{\eta}=\tilde{\eta}(\tilde{c})$ can be specified over the whole available range of concentration. Similarly for $M \leqslant M_{\text {erit. }}$ let $\tilde{\eta}_{\text {crit, }}$ be the maximum value of $\tilde{\eta}$ which still obeys the superposition principle.

An examination of the reduced viscosity functions, Figures 1 to 3, reveals the following features. At sufficiently low reduced concentrations, $\tilde{c} \leqslant 6$, say, superposition is obtained for all molecular weights considered. At larger $\tilde{c}$, differences between the polymer series become apparent. Consider P2VN and PVB where the $M$ range is similar. In the latter, increasing departures from the general relation with increasing $\tilde{c}$ occur for $M_{w}<M_{\text {crit. }} \simeq 169000$, whereas no systematic deviations are observable for the former. The molecular weight range of PIVN studied is narrow, but deviations may be noted for the lowest fraction, $M_{w}=52200$. The concentration range, preparation and treatment of the solutions were identical and the solvent conditions at the single temperature used not too different. We may enquire therefore whether a correlation exists between the above differences and differences in the intrinsic chain conformations and solute-solvent interactions.

For this purpose consider the short and long range interaction parameters $\sigma$ and $B$ :

$$
\begin{aligned}
& \sigma=\left(\left\langle r_{\theta}^{2}\right\rangle /\left\langle r_{0 f}^{2}\right\rangle\right)_{N=\text { const. }}^{1 / 2} \\
& B=\left(1 / 4 \pi^{3 / 2}\right)\left(N^{2} \bar{\beta} / 2 M^{2}\right)
\end{aligned}
$$

where $\left\langle r_{\theta}^{2}\right\rangle$ and $\left\langle r_{0 f}^{2}\right\rangle$ are the mean square end-to-end distance of the unperturbed and freely rotating chain respectively, $N$ is the number of chain carbon atoms, $\bar{\beta}$ the double cluster integral and the other symbols have their usual meanings. The recently calculated $\sigma$ and $B$ values of $\mathrm{PVAr}^{7}$ and PS at $30^{\circ} \mathrm{C}$ are shown in Table 5.

Table 5. Solution parameters of PVAr and PS

| No. | Polymer | Solvent | $\begin{gathered} \text { Temp. } \\ { }^{\circ} \mathrm{C} \end{gathered}$ | $\begin{gathered} K \times 10^{4} \\ \mathrm{~d} 1 / \mathrm{g} \end{gathered}$ | $a$ | $\boldsymbol{K}_{1}{ }^{\prime}$ | $a_{1}$ | $\begin{gathered} C \\ e q .(4) \end{gathered}$ | $\underset{\text { Mrit }}{M_{\text {crit }}}$ | $N_{\text {crit. }}$ | $\sigma^{*}$ | $\begin{gathered} B^{*} \\ \left(\times 10^{21}\right) \end{gathered}$ $\mathrm{cm}^{\mathrm{a}}$ | $\sigma^{*} C$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | P1VN | benzene | 30 | $1 \cdot 22$ | $0 \cdot 67$ | 849 | $0 \cdot 56$ | $0 \cdot 122$ | >52.2 | $>678$ |  |  |  |
| 2 | P2VN | benzene | 30 | $1 \cdot 48$ | 0.66 | 481 | 0.50 | $0 \cdot 143$ | $\ddagger$ | $\ddagger$ | 2.95 | 0.47 | $2 \cdot 10$ |
| 3 | PVB | benzene | 30 | 2.95 | 0.59 | 639 | 0.53 | $0 \cdot 101$ | 169 | 1880 | $2 \cdot 63$ | 0.35 | 1.81 |
| 4 | PS $\dagger$ | cyclo- | $34 \cdot 4$ | $9 \cdot 02$ | $0 \cdot 50$ | 260 | 0.47 | $0 \cdot 170$ | $<15$ | $<288$ | $2 \cdot 20$ | $0 \cdot 00$ | 1.81 |
| 5 | PS $\dagger$ | toluene | 30 | $1 \cdot 18$ | $0 \cdot 71$ | 1955 | 0.64 | 0.196 | 15 | 288 | $2 \cdot 20$ | $1 \cdot 76$ | 1.96 |

tef. 7
lelow range studied
In Table 5 we also show the value of $M_{\text {orit. }}$ which is a measure of the validity of our superposition method; the higher the value of $M_{\text {crit. }}$ the less applicable the method for the polymer-solvent system.

No results are given for P1VN. The calculation based on a linear plot of $[\eta] / M^{1 / 2}$ versus $M^{1 / 2}$ yields $\sigma=1.8$ when the molecular weights in Table 1 are used. This value is unreasonably small. For P2VN and PVB $M_{w}$ s, determined by GPC and light scattering ${ }^{4}$ respectively, yield $\sigma$ s which agree within two per cent, provided the lowest molecular weight for P 2 VN and the highest for PVB are omitted. These two values deviate markedly from the linear relationship. On the other hand, for PIVN $M_{w}$ from light scattering leads to $\sigma=2.4$. This is reasonable but at complete variance with the result derived from GPC. The narrow range of molecular weights does not permit further analysis.

Because of the limited number of fractions we cannot establish precise numerical values of $M_{\text {crit. }}$. However, a parallelism between $M_{\text {orit. }}$ and $\sigma$ is noticeable in Table 5. For the present agreement only the relative order, but not the absolute magnitude of $\sigma$ is pertinent. An increase of $\sigma$ results in a larger $M_{\text {crit. }}$. That is, departures from superposition at elevated concentrations already appear at relatively elevated molecular weights. This conclusion is not altered by the use of chain lengths $N_{\text {crit. }}$ in Table 5, since they also appear to be extremely sensitive to changes in $\sigma$ in the vinyl aromatic series. Comparisons should strictly be made at closely similar long range interactions. $\quad M_{\text {crit. }}$ is apparently less sensitive to variations in $B$ than in $\sigma$, but studies of polyvinyl aromatics in theta solvents and of very low molecular weights ( $M<15000$ ) of PS in cyclohexane are desirable. For the latter polymer we note a decrease of $M_{\text {crit }}$. with decreasing $B$ and recall analogous observations for other polymers ${ }^{3}$.

Not surprisingly, of course, the parameter $\sigma$ is not sufficient to characterize the limits of superposition in widely different polymer systems. For example, for polyisobutylene ${ }^{8} \sigma=1 \cdot 8$, whereas $N_{\text {crit. }} \simeq 360$ in isooctane.

We have referred to the range of molecular weights $M>M_{\text {crit. }}$ over which superposition applies. It is noteworthy also that in the polyvinyl aromatic
series the departures occur in a considerably lower range of reduced viscosities than for other polymers ${ }^{3}$, where $\tilde{\eta}_{\text {crit. }}$ exceeds at least five.
(2) The concentration parameter $\gamma^{\prime}$

In a suspension of hard spheres the obvious measure of the reduced concentration is $\phi / \phi_{\mathrm{cp}}$, where the $\phi s$ are volume fractions and the subscript 'cp' signifies close packing. Hence by analogy $\gamma^{\prime}$, equation (1), may be considered to be proportional to an effective packing volume of the polymer coils. As for the molecular weight dependence, the inequality $a>a_{1}$ has been observed earlier in good solvents ${ }^{1-3}$. Table 5 indicates a similar relationship for the polyvinyl aromatics studied here.

The absolute magnitude of $\gamma$ and hence of $C$, equation (4), depends on the arbitrary choice of a reference molecular weight. The dimensionless parameter $C$, of course, is independent of molecular weight. Judging from the not too different initial slopes of curves 4 and 5, Figure $4^{1}$, and Table 5,


Figure 4-Semilogarithmic plot of reduced viscosity quantity $\tilde{\eta}$ as a function of reduced concentration $\tilde{c}$ for PS-cyclohexane: $T=\theta$ (curve 4) ${ }^{1}$, PS-toluene $30^{\circ} \mathrm{C}$ (curve 5), P1VN-benzene $30^{\circ} \mathrm{C}$ (curve 1), P 2 VN -benzene $30^{\circ} \mathrm{C}$ (curve 2), PVB-benzene $30^{\circ} \mathrm{C}$ (curve 3)
$C$ is weakly dependent on solvent power, but this observation deserves further study. Again, there is a more pronounced variation with $\sigma$. It is suggestive to treat $C$ as a ratio between two characteristic coil volumes and
to consider its dependence on $\sigma^{3}$. In Table 5 it is seen that the product $C \sigma^{3}$ is reasonably constant. No significant improvement of this constancy would result from the use of a viscometric coil volume and the quantity $\sigma^{3} \alpha^{n}$, where $n \leqslant 3$ and $\alpha$ represents the polymer coil expansion factor. This may be another indication that the effective packing volume in our concentration range is principally determined by the unperturbed coil dimensions and only secondarily by polymer-solvent interactions.

## (3) Relative position of reduced curves

The master curves, $\tilde{\eta}=\tilde{\eta}(\tilde{c})$, of the five polymer-solvent systems, based on $\gamma_{1}$, are plotted in Figure 4. Generally speaking, their relative positions are paralleled by the relative values of the conformational parameter $\sigma$, of $M_{\text {crit. }}$ and $M_{0}$. The variation with $M_{0}$ arises from the fact that for a given $M$ and hence $\gamma_{1}$, the chain length is smaller for the larger $M_{0}$. Since the dependence of $\eta_{\text {sp }} / c$ on chain length is greater than that of $[\eta], \tilde{\eta}$ is smaller. For constant $M_{0}$, we observed ${ }^{1-3}$ earlier an increase in the derivative $\mathrm{d} \tilde{\eta} / \tilde{\mathrm{d}} \tilde{c}$ at sufficiently large $\tilde{c}$ with decreasing solvent power; compare for example curves 4 and 5 (Figure 4). A similar pattern is indicated by P1VN and P 2 VN in benzene.

In an attempt to isolate the effect of long range interactions on the derivatives $\tilde{\mathrm{d}} \tilde{\eta} / \tilde{\mathrm{d}}$, we account for the differences in monomer mass and chain stiffness by the introduction of the reduced concentration

$$
\begin{equation*}
\tilde{c_{1}}=\left(c / M_{0}\right) /\left(\gamma \sigma^{3}\right)=\tilde{c} /\left(M_{0} \sigma^{3}\right) \tag{6}
\end{equation*}
$$

The placement of the factor $\sigma^{3}$ is consistent with the approximate constancy of $C \sigma^{3}$ in equation (4). If $\gamma$ is normalized by the chain length $N_{1}=M_{1} / M_{0}, \gamma\left(N_{1}\right)=1$, then $\gamma \equiv \gamma_{1}$. Consider now the comparative values of ( $\tilde{c_{1}} \times 10^{2}$ ) for a fixed value of $\tilde{\eta}$, say, 3.5 in Figure 4. They are in decreasing order $1.60,1 \cdot 25,1.22$ and $1 \cdot 18$ for the systems $5,3,4,2$ respectively. The corresponding values of ( $B \times 10^{27}$ ) in Table 5 are 1.76, $0.35,0.00$ and 0.47 . The order of the two quantities is the same except for system 2, P2VN-benzene, where we should have expected $0.0125<\widetilde{c_{1}}(\mathrm{P} 2 \mathrm{VN})<0.0160$. Actually, were we to put $\sigma=2.40$, the value derived from the light scattering $M_{w}$, instead of $\sigma=2 \cdot 45, c_{1}$ would be increased to $0 \cdot 0126$.

Certainly, the suggestion made here in connection with equation (6) requires scrutiny by means of accurate $\sigma$ and $B$ determinations on these and other polymer-solvent systems.

## CONCLUSION

The range of applicability of the proposed method of superposition of viscosity/concentration data is limited for polyvinyl aromatics compared with that for polystyrene. The limiting factors seem to be mainly the chain stiffness indicated by the magnitude of the conformational parameter $\sigma$ for a given series of polymers and to a lesser extent polymer-solvent interactions. An increase in chain extension, due to either the first or the second factor, results in an increase of the limiting molecular weight above which a
principle of corresponding states is obeyed over the whole range of concentration explored here. It appears that $M_{\text {crit. }}$ and the effective packing volume of the polymer coils within a homologous series as derived from $\gamma$, depend mainly on short range interactions. In this context it would be highly desirable to examine systems containing stiff polymer chains, for example 1,4 -polydienes, ribbon and ladder type polymers, polymers with aromatic groups in the main chain, etc. Unfortunately there are no such data available in the literature.

The effects of the short and long interaction parameters on the position of the general curves (for $M \geqslant M_{\text {crit. }}$.) can be separated by using a reducing concentration $\widetilde{c_{1}}$ defined in equation (6). An increase of $B$ tends to decrease $\tilde{\eta}$ for the same $\tilde{c_{1}}$.

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